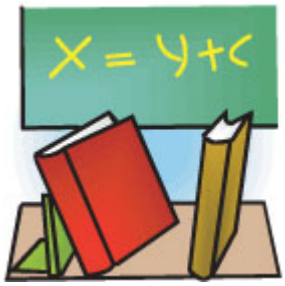
Diocese of Trenton

Mathematics Curriculum Guidelines



Algebra 1

September 2011

Algebra I Program in the 8th Grade

Algebra and algebraic reasoning remain at the core of the high school mathematics curriculum. Algebra's dominance in the school curriculum is related to the importance of algebra in more advanced areas of mathematics, the usefulness of algebraic reasoning in all walks of life, and the role of algebra as a tool for the mathematical modeling required in many technological and scientific fields**.** (Introduction to Focus in High School Mathematics: Reasoning and Sense Making in Algebra, NCTM, 2009)

The Common Core Educational Standards set a protocol for teaching algebra from Pre-K-12 as well as naming standards for an algebra course. The CCES states that coherence is essential in teaching mathematics by aligning curriculum, instruction and assessment. The CCES also state Mathematical practices that are fundamental to the teaching and learning of mathematics:

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

N.B. See Pre-K-8 introduction for extended detail of the practice standards.

Algebra I can be offered to 8th grade students if a school has a teacher who is qualified to teach Algebra I and specific criteria is developed to admit students to this program. Algebra I is a high school course that is offered to middle school students and the students must be capable of succeeding in a high school course. Offering Algebra I to 8th grade students must be carefully considered by the principal, teacher and student.

If a school states that they offer Algebra I to 8th grade students then it is the responsibility of the school to ensure that the students complete the High School Algebra curriculum so that the students can successfully advance in the high school mathematics curriculum.

The following criteria are critical for success and admitting students to an Algebra I program in the 8th grade:

- ❖ Earn 85% or greater in Pre-algebra classroom work
- ❖ Earn 85% or greater in the end of course test for pre-algebra in the 7th grade
- ❖ Score in the proficient category in Mathematics in the Terra Nova testing process
- ❖ Score 90% or above on the Terra Nova Math test
- ❖ Expresses interest in advancing in Mathematics
- ❖ Parent permission to take Algebra I
- ❖ Teacher recommendation

It is expected that 8th grade teachers meet periodically with the receiving High School mathematics department to fully understand the expectations of high school mathematics. It is recommended that all 8th grade teachers familiarize themselves with the mathematics problems in PSAT and SAT examinations. This is important because some 8th grade Algebra I students will move into Geometry and never take Algebra I again.

It is recommended that every Algebra teacher in Middle School and High School read the NCTM publication, ***Focus in High School Mathematics, Reasoning and*** Sense making in Algebra, (NCTM, 2009). This current publication stresses that students of mathematics focus on reasoning and understanding how mathematics works…in other words develop a sense of mathematics. Teachers need to shift to facilitating mathematics from dictating mathematics.

Algebra I programs should be offered to all 8th grade students who qualify and can be successful in Algebra I. A summative Algebra assessment should be administered to very 8th grade student who completes the Algebra I program in the 8th grade.

Placement of 8th Grade Algebra Students in Secondary School

The 8th grade teacher and the secondary school personnel should collaborate concerning the placement of the 8th graduates in the secondary school mathematics programs.

Key Symbols for Standards

CCSM – Common Core Standards in Mathematics
DTM – Diocese of Trenton Mathematics
AS – Algebraic Structure
SAE – Simplifying Algebraic Expressions
PRE – Polynomial and Rational Expressions
SEI – Solving Equations and Inequalities
EXP – Exponents
SOE – Systems of Equations and Inequalities
GEI – Graphing Equations and Inequalities
FN – Functions
MM – Mathematical Modeling
*** - enrichment or advanced**

The Importance of Using Algebra in Mathematical Modeling

Mathematical modeling is the process of using various mathematical structures - graphs, equations, inequalities, diagrams, and so forth - to represent real-life situations. The model provides an abstraction that reduces a problem to its essential characteristics. The essential characteristics can be altered and manipulated symbolically to arrive at a conclusion or solution in its easiest form.

Mathematical modeling ***must*** permeate the entire course of study while teaching algebra. It is not an entity in and of itself. Therefore, applications using algebraic symbolism and structure ***must*** be inserted as each topic of algebra is introduced. A continuous and gradual growth using mathematical modeling will assist students in becoming competent problem solvers and critical thinkers using and applying higher order thinking skills (HOTS) in conjunction with mathematical structures.

N.B. If graphing calculators are not available for students access Microsoft Mathematics (free software) on the internet as a substitute for the graphing utility. www.microsoftmathematics.com/download

Essential Questions:

- Why is it important to solve equations in a specific order?
- How do you determine the method that is most appropriate for solving a system of equations?
- How is the study of Algebra connected to the study of Language Arts?
- Using the concepts of Algebra solve a problem without the use of numerical or algebraic symbolism.
- How is Algebra connected to other Mathematical disciplines?
- Describe how Algebra is necessary for the study of Science.
- How is Algebra used in the construction of a home?
- How is mathematical symbolism used in the Bible?
- Research the origins of the Pythagorean Theorem and prove its concepts.

CONCEPT	**CONTENT**
Structure and Algebraic Expression (AS-DTM and SAE-DTM)	Definition of variable and variable expression
	Understand the difference between term, factor, and coefficient ***(CCSM)***
Concept Development	***Understanding the structure of a Mathematical System***
	Properties of the Real Number System also called Field Axioms: Closure, Commutative, Associative, Distributive, Identity and Inverse ***(CCSM)***
Concept Development	***Understanding the link between Arithmetic and Algebra***

	Translate from words to algebraic symbols
	Substitution Principle
	Evaluating Algebraic Expressions ***(CCSM)***
	Using Grouping symbols (Symbols of Inclusion: Parenthesis, Brackets, Braces, Bar)
	Recognize equivalent algebraic expressions ***(CCSM)***
	Interpret and apply the use of signed numbers.
	Locating signed numbers on the number line (Property of Density).
	Operations of signed numbers (Illustrate addition and subtraction on the number line.)
	Apply signed numbers to real-life applications
	Using order of operations with Algebraic Expressions
	Expressing Rational Numbers in various formats
	Graphing Rational Numbers on the number line
	Intuitive definition of absolute value
	Symbolic (formal) definition of absolute value.
Solving Equations (SEI-DTM)	Solving equations intuitively
	Solve one-step equations involving each of the four operations (+,-,×,÷).
	Solve multistep equations (include whole number, integer, fraction and decimal coefficients)
	Utilize the Properties of Equality (+,-,×,÷). Transformations in solving multistep equations.
	Solve equations with variables on both sides
Concept Development	***Explain transformations using the Field Axioms.*** **(CCSM)**
	*Using proofs in Algebra(AS)
	Solving Literal Equations **(CCSM)**
	Using equation in mathematical modeling (e.g. number problems, consecutive integer problems)
Solving Inequalities (SEI-DTM)	Define the meaning of the symbols (<,>,≥,≤) Less than, greater then, and equal to
Concept Development	***Introduce the meaning of terms conjunction and disjunction;*** ***Trichotomy Property; Transitive Property of Order***
	Solve one-step and multi-step inequalities Graph solutions on the number line

	Solve inequalities with negative leading coefficient
	Solve absolute value inequalities (Graph solutions on the number line)
Exponents (EXP-DTM)	Write and simplify expressions using exponents
	Introduce and apply negative exponents
Concept Development	**$a^0=1$, $a\varepsilon\mathbb{R}$, $a\neq0$**
	Perform calculations using scientific notation
Concept Development	***Present the properties of exponents in symbolic form***
Polynomials and Rational Expressions (PRE-DTM)	Define and Combine Similar Terms
	Addition and Subtraction of Polynomial Expressions
	Powers of Monomials
	Multiplying Polynomials by Monomials (Use of the Distributive Property)
	Multiplying polynomials [Use Distributive Property and Vertical Multiplication (to compare to multi-digit arithmetic multiplication)]
	Multiplying Binomials (FOIL and mental multiplication)
Mathematical Modeling (MM-DTM and *CCSM*)	Uniform Motion Problems, Area applications, (*Problems ***without*** solutions or insufficient information)
	Squaring Binomials (paper and pencil and mentally)
	Finding LCM and GCF algebraically.
	Dividing Monomials
Concept Development	***Definition of Division***
	Divide polynomials by monomials
	Finding monomial factors of polynomials
	Dividing polynomials by binomials
Mathematical Modeling (MM-DTM and *CCSM*)	Finding area of shaded region
	Factoring the Difference of 2 Perfect Squares
	Factoring the sum or difference of 2 perfect cubes (* $a^3 + b^3$ or $a^3 - b^3$)
	Factoring trinomials of the form $x^2 + bx + c$ (c positive or negative)
	Factoring polynomials of the form $ax^2 + bx + c$
	Factoring by Grouping
	Using multiple methods of factoring (factoring completely)

	Solving equations by factoring
Concept Development	***Discuss the Zero-Product Property (symbolically)***
	Solving Quadratic Equations by factoring and the quadratic formula with rational number solutions (*completing the square)
Mathematical Modeling (MM-DTM and *CCSM)*	Solving problems involving writing and factoring polynomials
	Simplifying Rational expressions using factoring
	Multiplying and Dividing Rational Expressions
	Finding LCM of denominators using factoring
	Addition and Subtraction of Rational Expressions
	Adding and Subtracting Mixed Algebraic Expressions
Concept Development	***Relationship between arithmetic and algebraic mixed fractions*** $a + \frac{b}{c} = \frac{ac + b}{c}$
	*Simplifying complex fractions
	Solving equations with fractional coefficients
	Solving Fractional Equations
Mathematical Modeling (MM-DTM)	Investigate and solve percent application problems
	Investigate and solve Mixture Problems, Coin Problems, Electrical Resistance and Work Problems
Equations in Two Variables (GEI+DTM)	Standard form of a linear equation $Ax + By = C$
	Finding slope and y-intercept using standard form
	Slope of a line given two points $m = \frac{y_2 - y_1}{x_2 - x_1}$
	Slopes of horizontal line =0 Slope of a vertical line is undefined (no slope)
	Slope-intercept form of a linear equation: $y = mx + b$
	Point-slope form of a linear equation: $y - y_1 = m(x - x_1)$
	Intercept form of a linear equation: $\frac{x}{a} + \frac{y}{b} = 1$; a = x-int. and b = y-int.*
	Parallel lines have the same slope Slopes of perpendicular lines are opposite reciprocals OR the product of their slopes is -1
	Determining the equation of a line give:

	a) slope and y-intercept b) two points c) point and slope d) *x- and y-intercepts
Functions (FN+DTM and CCSM)	Define ***relation***, ***function, domain*** and ***range***
	Introduce and utilize function notation
Concept Development	***Understand that in a function, each element of the domain is paired with exactly one element in the range. (CCSM)***
	Represent functions in various formats: a) using an equation b) using function notation c) graphing in the coordinate plane
	Graphing linear and quadratic functions (*exponential functions)
	Graphing quadratic functions (*using the axis of symmetry and vertex)
Mathematical Modeling (MM+FN+DTM)	Applications of functions that arise from different contexts ***(CCSM)***
Systems of Linear Equations (SOE + DTM and ***(CCSM)***	Solving systems of equations using : a) graphing b) substitution c) linear combination d)* matrices
Concept Development	****Consistent or Inconsistent Systems of Equations AND Independent or Dependent System***
Mathematical Modeling (MM+SOE+DTM)	Application involving systems or equations (e.g. Finance, Wind and Water Currents)
Graphing Linear Inequalities in Two Variable	Graphing inequalities involving (<, >, ≤, ≥)
Graphing Systems of Inequalities	Graphing 2 or more inequalities in the same coordinate plane
	*Linear Programming Application
The System of Real Number	Define Rational Number and Irrational Number
Concept Development	***Property of Density for Rational Numbers*** ***Property of Density for Real Numbers***
	Various form of Rational Numbers
	Various forms of Irrational Numbers
	Simplifying Numerical Radical Expressions
Concept Development	***Multiplication and Division Properties of Radicals expressed symbolically***
	Multiplying and Dividing Numerical Radical Expressions

	Addition and Subtraction of Numerical Radical Expressions
	Operations of Radical expressions with variables
Mathematical Modeling	The Pythagorean Theorem
	*Distance and Midpoint formula
Solving Equations	Solving quadratic equations using the quadratic formula with rational and irrational solutions (*completing the square)
	*Introduce complex numbers (real+imaginary)
Concept Development	***Using the discriminant to determine the nature of the roots of a quadratic equation.***
Mathematical Modeling	Applications of Quadratic Equations
Mathematical History	Research the origins of Algebra